SHORE

Examination Number:

Set:

2023 YEAR 12
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent reviewing your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

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1 If a function is a continuous probability distribution, what is the area under the curve?

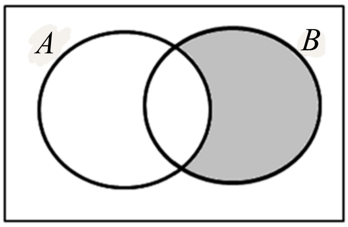
A. -1

B. 0.5

C. 1

D. 2

2 Two sets A and B are represented in the Venn diagram below.



The shaded region can be described by which of the following?

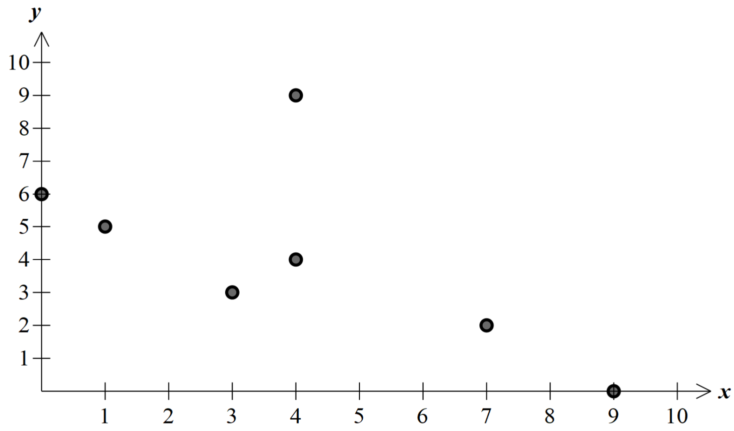
A. B

B. $A \cap B$

C. A'

D. $A' \cap B$

3 The correlation coefficient for the scatterplot shown was found to be -0.6.



The point (4, 9) was found to be recorded incorrectly and should have been plotted as (4, 1).

Which of the following best describes the new correlation coefficient?

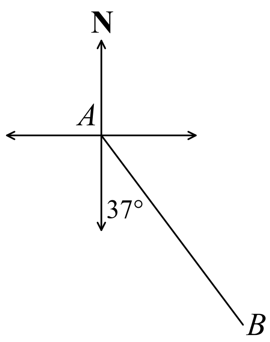
A. Positive but closer to 0

B. Positive but closer to 1

C. Negative but closer to 0

D. Negative but closer to -1

4 Consider the diagram below.



What is the true bearing of A from B?

A. 037°

B. 143°

C. 307°

D. 323°

5 The probability distribution table for a discrete random variable X is shown.

x	1	2	3	4
$P(X=x)$	0.4	0.2	0.15	0.25

What is the expected value of X ?

A. 0.4

B. 1.0

C. 1.5

D. 2.25

6 What is the exact value of $\int_{-6}^{0} \sqrt{36-x^2}\,dx$?

A. 6

B. 9

C. 6π

D. 9π

7 Packets of Cereal are labelled as having a mass of 500 grams. The machine that fills the packets follows a normal distribution with a mean of 510 grams and a standard deviation of 5 grams.

What percentage of packets will have a mass less than 500 grams?

A. 2.5%

B. 5%

C. 34%

D. 50%

8 Which of the following is the gradient of the normal to $y = \log_3 x$ at the point $(9, 2)$?

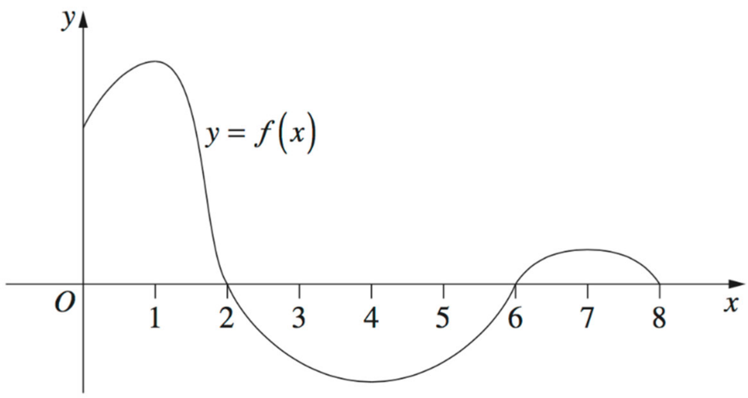
A. $-\dfrac{1}{9\ln 3}$

B. $-9\ln 3$

C. $\dfrac{1}{9\ln 3}$

D. $9\ln 3$

9 The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$.



Which of the following integrals has the smallest value?

A. $\int_0^1 f(x)\,dx$

B. $\int_0^2 f(x)\,dx$

C. $\int_0^6 f(x)\,dx$

D. $\int_0^8 f(x)\,dx$

10 For what values of m does the quadratic equation $x^2 + mx + (m+1)^2 = 0$ have two equal roots?

A. $m = \frac{2}{3}$, $m = 2$

B. $m = \frac{2}{3}$, $m = -2$

C. $m = -\frac{2}{3}$, $m = 2$

D. $m = -\frac{2}{3}$, $m = -2$

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet **A**

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

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Question 11 (2 marks)

Find the centre and radius of the circle with the equation $x^2 + 4x + y^2 - 8y - 5 = 0$. **2**

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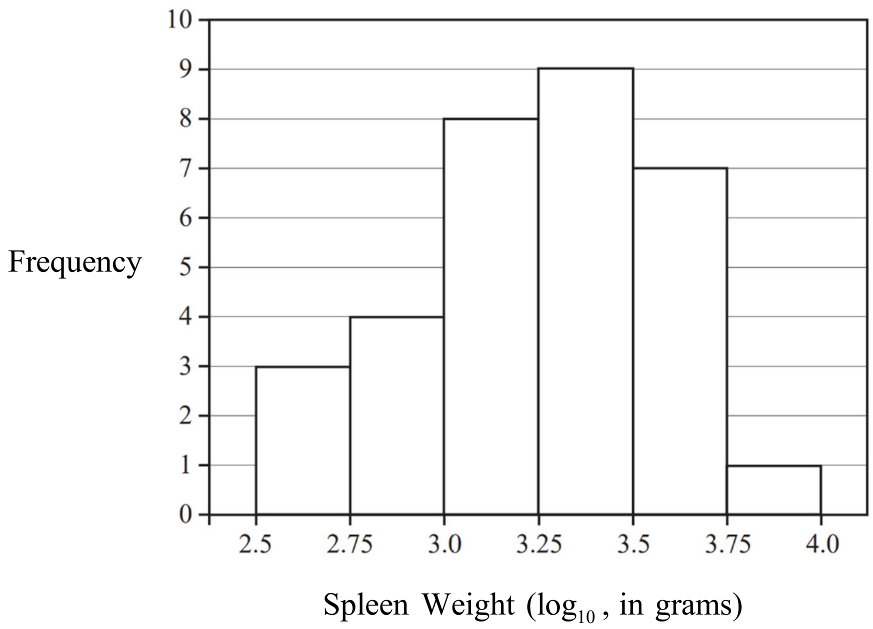
Question 12 (1 mark)

The histogram below shows the distribution of spleen weight for a sample of 32 seals. **1**
The histogram has a $\log_{10}$ scale.



Find the number of seals in this sample with a spleen weight of more than 1000 grams.

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Question 13 (5 marks)

The table shows the height and weight of basketball players on the 2013 roster for the NBL Perth Wildcats.

Height H (cm)	191	201	200	204	211	192	196	203	188	202	186
Weight W (kg)	92	99	95	109	105	97	95	100	82	103	92

(a) Using your calculator, find the correlation coefficient for this data correct to two decimal places. **1**

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(b) Describe the correlation between height and weight. **1**

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(c) Find the equation of the least-squares regression line for this data. **1**

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(d) Use the equation in part (c) to estimate the weight of a player that is 198 cm tall. Answer to 1 decimal place. **1**

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(e) Can your equation from part (c) be used to make reliable estimates for a player who is 162 cm tall? Justify your response. **1**

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Question 14 (1 mark)

If $\int_2^6 g(x)\ dx = 10$, determine the value of $\int_2^6 \left(g(x)+3\right)\ dx$ given $g(x) > 0$. **1**

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Question 15 (3 marks)

The tangent to the graph of $f(x) = x^3 - ax^2 + 1$ at $x = 1$ passes through the origin. **3**

Find the value of *a*.

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Question 16 (5 marks)

Sophie retires from being a teacher, with a superannuation balance of \$775 320. She decides on receiving a monthly annuity of \$7000, with 7.5 % per annum interest paid on the balance, before the annuity is paid at the end of each month.

The annuity can be modelled by the recurrence relation

$$T_{n+1} = (1+r)T_n - 7000, \quad T_0 = 775320$$

where T_n represents the balance in the superannuation fund after n months.

(a) Show that $r = 0.00625$. **1**

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(b) After 3 months, what is the balance remaining in the fund? **2**

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(c) Sophie received \$7000 per month for 189 months, in addition to a single payment of \$1000.40 to bring the balance to zero. Determine the interest component of the total amount Sophie received. **2**

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet B

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

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Please turn over

Question 17 (7 marks)

Consider a random variable X with a probability density function defined by:

$$f(x) = \begin{cases} kx^2 & 2 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{1}{168}$. **2**

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(b) Find the mode. **2**

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(c) Find the median correct to two decimal places. **2**

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(d) Given the mean is $x = 6.07$, describe the skew of the distribution. Justify your answer. **1**

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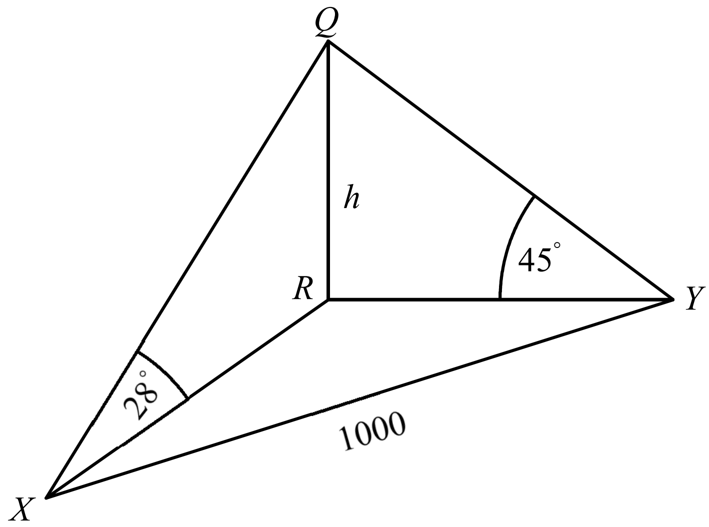
Question 18 (5 marks)

The angle of elevation of a tower QR of height h metres from a point Y due east of it is 45°. From another point X, the bearing of the tower is 030° and the angle of elevation is 28°. The points X, Y and R are on the same level ground. The distance between X and Y is 1000 metres.



(a) Show that $\angle XRY = 120^{\circ}$. **1**

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(b) Find an expression for XR. **1**

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(c) Hence find the height of the tower QR. Give you answer to the nearest metre. **3**

Question 19 (2 marks)

Solve $2 = 4\cos 3x$, in the interval $0 \le x \le \frac{\pi}{4}$. **2**

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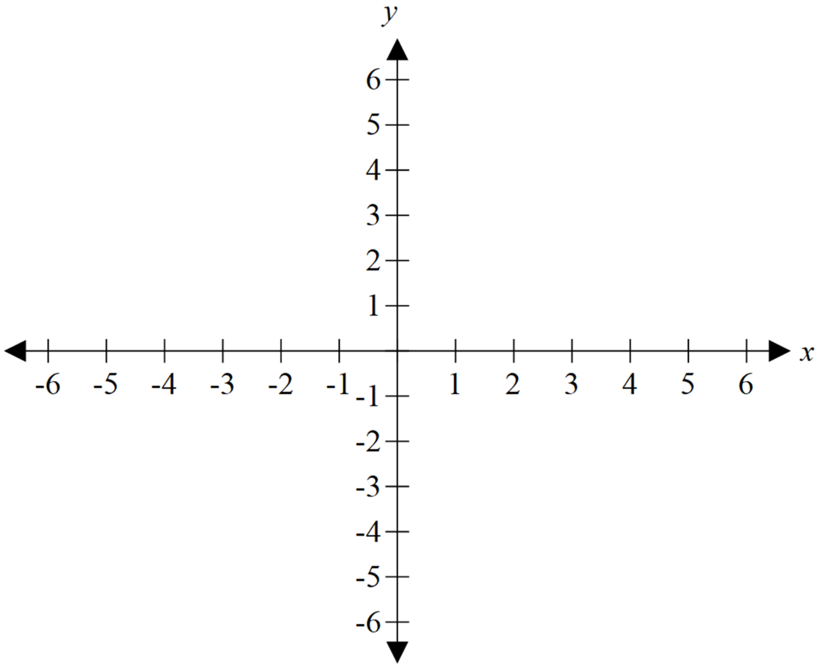
Question 20 (2 marks)

Sketch the graph $y = 1 - \frac{1}{x+3}$, showing asymptotes and the x and y intercepts. **2**



End of Booklet B

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet C

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

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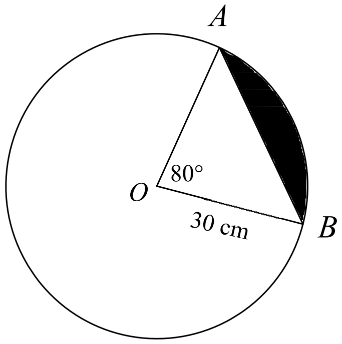
Question 21 (5 marks)

The diagram shows a circle with centre O and radius 30 cm. The points A and B lie on the circle such that $\angle AOB = 80^\circ$.



(a) Convert 80° to radians. **1**

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(b) Show that the area of sector AOB is equal to $628\,\text{cm}^2$, rounded to the nearest square centimetre. **1**

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(c) Find the perimeter of the shaded segment, giving your answer correct to one decimal place. **3**

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Question 22 (3 marks)

Solve the equation $\ln(x^2 - 5) = 2\ln x - \ln 5$. **3**

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Question 23 (5 marks)

Professor Smith has a colony of bacteria. Initially there are 1000 bacteria.

The number of bacteria $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$.

(a) After 20 minutes there are 2000 bacteria. Find the value of k correct to four decimal places. **2**

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(b) Find the amount of bacteria when $t = 120$. **1**

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(c) What is the rate of change of the number of bacteria per minute, when $t = 120$? **2**

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Question 24 (3 marks)

600 students at a primary school were asked whether they preferred Rugby, Soccer or AFL. The results are given in the following table.

	Rugby	Soccer	AFL
Female	52	130	88
Male	104	155	71

A student is chosen at random.

(a) Find the probability that the student prefers Rugby. **1**

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(b) Determine whether the events "the student is Female" and "the student prefers Rugby" are independent, justifying your answer with mathematical reasoning. **2**

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Question 25 (2 marks)

The graph $y = f(x)$ of $f(x) = x^3$ is translated 3 units right and 5 units up, then horizontally dilated by a scale factor of $\frac{1}{2}$ to produce $y = g(x)$. Find the equation of the transformed function $g(x)$. **2**

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2023	YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet D

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

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Please turn over

Question 26 (4 marks)

Susannah is competing in a card building competition where the number of cards to build each level follows an arithmetic sequence. She builds the fifth level of the stack using 200 cards. For the first four levels, she uses a combined total of 1200 cards.

(a) Find the common difference between the number of cards used to build each level. **2**

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(b) How many cards were used to create the base level? **1**

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(c) How many levels will Susannah be able to make? **1**

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Question 27 (3 marks)

Show that $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta}-\dfrac{\sec\theta}{\tan\theta+\cot\theta}=\cos\theta$. **3**

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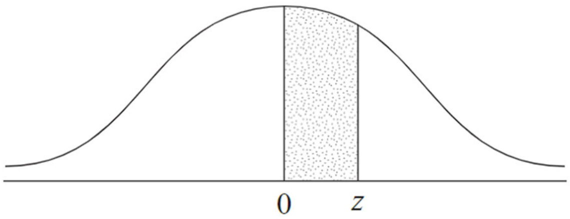
Question 28 (6 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table below shows the probability this random variable lies between 0 and z for different values of z.

z	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



(a) Using the table, find the probability that a value lies between 0.2 and 0.6. **1**

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Daily charges for gas usage are normally distributed with a mean of $7.65 and standard deviation of $1.44.

(b) Two adults are comparing their gas charges. Find the probability that both of their gas charges are between $6.21 and $10.53. **2**

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(c) By first calculating a *z*-score, find how many people out of 1000, are expected to have a daily charge greater than \$8.37. **3**

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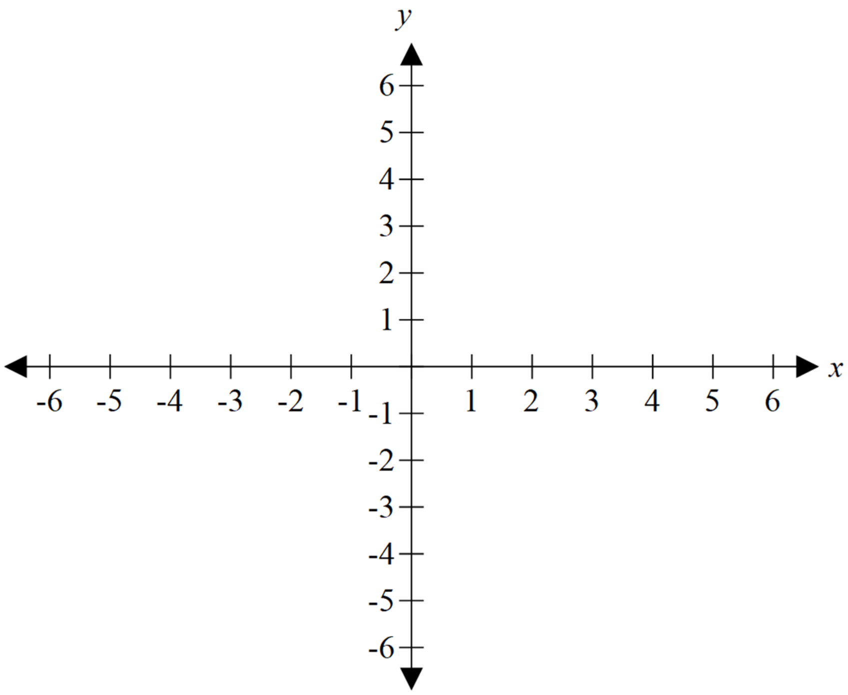
Question 29 (7 marks)

Consider the curve $y = 3x^2 + x^3$.

(a) Find the coordinates of any stationary point(s) and point(s) of inflection then determine their nature. **4**

(b) Sketch the curve $y = 3x^2 + x^3$, showing any stationary points, points of inflection and intercepts with the axes. **2**



(c) For what interval is the curve $y = 3x^2 + x^3$ decreasing? **1**

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:			
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Mathematics Advanced

Section II Answer Booklet E

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

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Please turn over

Question 30 (3 marks)

A particle moves in a straight line with an initial displacement equal to 1 metre and initial velocity equal to $1\ \text{m}\,\text{s}^{-1}$. Find the exact position of the particle after 5 seconds if its acceleration in $\text{m}\,\text{s}^{-2}$ is given by $a = 2\sin t$. Give your answer to the nearest metre. **3**

Question 31 (4 marks)

(a) Show that $\frac{d}{dx}\left(\log_e(\sin x)\right) = \cot x$. **1**

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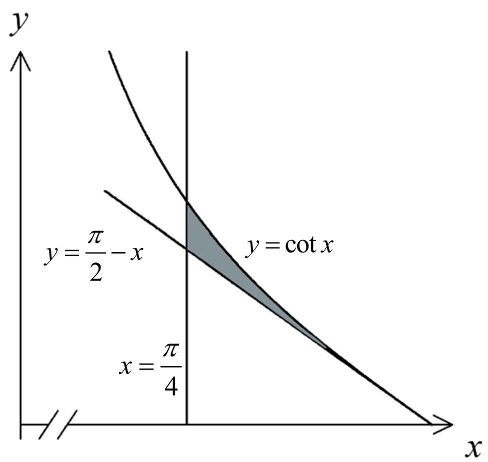
(b) The shaded region in the diagram is bounded by the curve $y = \cot x$ and the lines $y = \frac{\pi}{2} - x$ and $x = \frac{\pi}{4}$. Using the result of part (a), or otherwise, find the exact area of the shaded region. **3**



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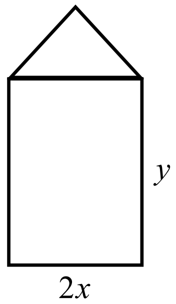
Question 32 (5 marks)

A stained glass feature is being created in the shape of a rectangle surmounted by an isosceles triangle of height equal to half its base. The perimeter is to be 150 cm. **5**



By showing that the area, in square centimetres, of the stained glass figure is given by $A = 150x - (2\sqrt{2} + 1)x^2$. Determine the width and the height of the figure for which the area is the greatest. Give your answer to one decimal place.

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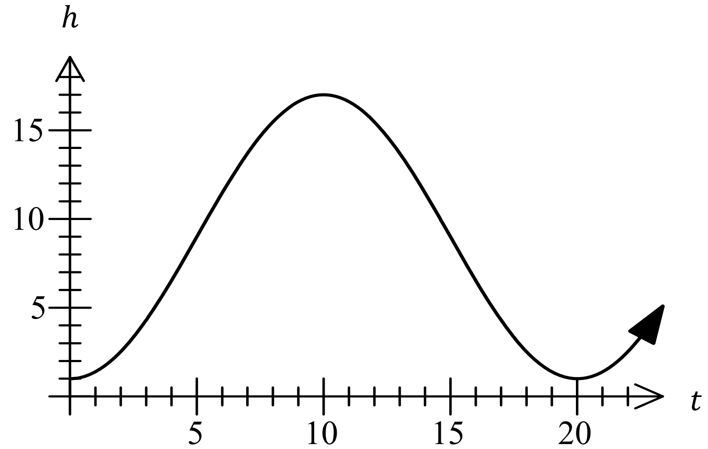
Question 33 (7 marks)

On a Ferris Wheel at a fair, the height of a carriage from the ground is modelled by the function $h(t) = -a\cos\left(\frac{\pi t}{10}\right) + b$, where t is the number of seconds after the ride has started, h is the height in metres and a and b are constants. The carriage has a maximum height of 17 metres when $t = 10$ and a minimum height of 1 metre when $t = 20$.

The graph of $h(t)$ is shown.



(a) What are the values of a and b? **2**

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(b) If the ride lasts for 3 minutes. How many rotations are completed before the ride comes to a stop? **2**

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(c) When the carriage is 11 metres or higher above the ground passengers can see the ocean in the distance. On a 3-minute ride, for how long do they see the ocean to the nearest second? **3**

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Section II extra writing space

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SHORE

2023

YEAR 12
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent reviewing your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

MC

1. C
2. D
3. D
4. D
5. D
6. D
7. A
8. B
9. C
10. D

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

---

1 If a function is a continuous probability distribution, what is the area under the curve?

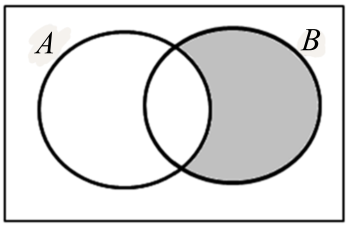
A. -1

B. 0.5

C. 1

D. 2

2 Two sets A and B are represented in the Venn diagram below.



The shaded region can be described by which of the following?

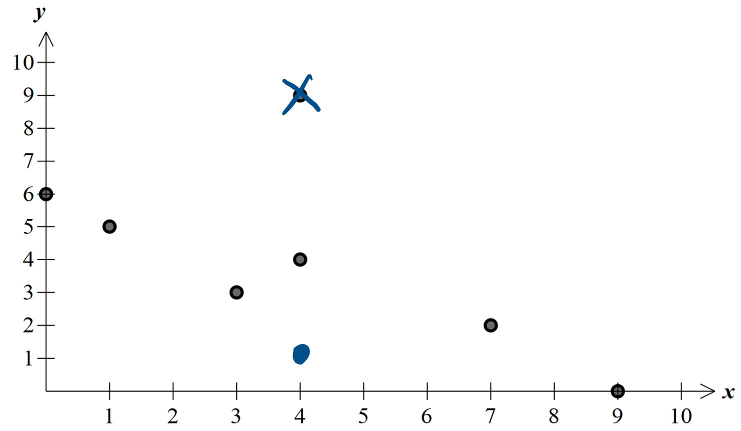
A. B

B. $A \cap B$

C. A'

D. $A' \cap B$

3 The correlation coefficient for the scatterplot shown was found to be -0.6.



The point (4, 9) was found to be recorded incorrectly and should have been plotted as (4, 1).

Which of the following best describes the new correlation coefficient?

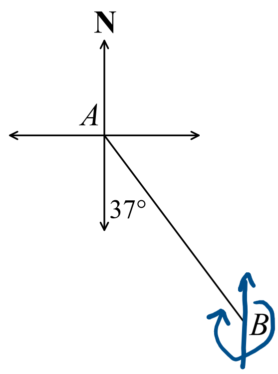
A. Positive but closer to 0

B. Positive but closer to 1

C. Negative but closer to 0

D. Negative but closer to -1

4 Consider the diagram below.



360 - 37

What is the true bearing of A from B?

A. 037°

B. 143°

C. 307°

D. 323°

5 The probability distribution table for a discrete random variable X is shown.

x	1	2	3	4
$P(X=x)$	0.4	0.2	0.15	0.25

What is the expected value of X ?

$(1\times 0.4)+(2\times 0.2)+(3\times 0.15)+(4\times 0.25)$

A. 0.4

B. 1.0

C. 1.5

D. 2.25

6 What is the exact value of $\int_{-6}^{0}\sqrt{36-x^2}\,dx$?

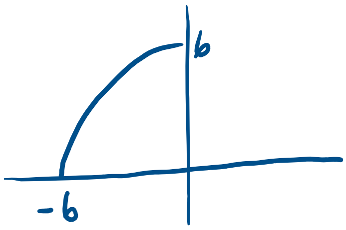
A. 6

B. 9

C. 6π

D. 9π

$\frac{\pi\times 6^2}{4}=9\pi$



7 Packets of Cereal are labelled as having a mass of 500 grams. The machine that fills the packets follows a normal distribution with a mean of 510 grams and a standard deviation of 5 grams.

What percentage of packets will have a mass less than 500 grams?

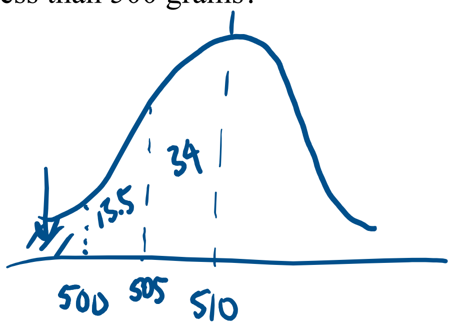
A. 2.5%

B. 5%

C. 34%

D. 50%

$50-34-13.5=2.5$



8 Which of the following is the gradient of the normal to $y = \log_3 x$ at the point $(9, 2)$?

A. $-\dfrac{1}{9\ln 3}$

B. $-9\ln 3$

C. $\dfrac{1}{9\ln 3}$

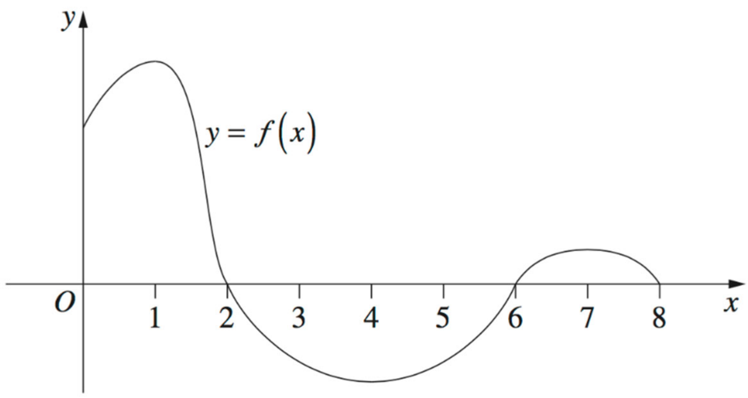
D. $9\ln 3$

$\dfrac{dy}{dx} = \dfrac{1}{x\ln 3}$

At $x=9$, $\dfrac{dy}{dx} = \dfrac{1}{9\ln 3}$

Normal, $m_2 = -9\ln 3$

9 The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$.



Which of the following integrals has the smallest value?

A. $\int_0^1 f(x)\, dx$

B. $\int_0^2 f(x)\, dx$

C. $\int_0^6 f(x)\, dx$

D. $\int_0^8 f(x)\, dx$

10 For what values of m does the quadratic equation $x^2 + mx + (m+1)^2 = 0$ have two equal roots?

A. $m = \frac{2}{3}$, $m = 2$

B. $m = \frac{2}{3}$, $m = -2$

C. $m = -\frac{2}{3}$, $m = 2$

D. $m = -\frac{2}{3}$, $m = -2$

$\Delta = 0$

$\Delta = b^2 - 4ac$

$\therefore \Delta = m^2 - 4(1)(m+1)^2$

$= m^2 - 4(m^2 + 2m + 1)$

$= m^2 - 4m^2 - 8m - 4$

$= -3m^2 - 8m - 4$

Let $\Delta = 0$

$3m^2 + 8m + 4 = 0$

$m = \frac{-8 \pm \sqrt{64 - 4(3)(4)}}{2(3)}$

$= \frac{-8 \pm \sqrt{16}}{6}$

$= -2, -\frac{2}{3}$

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet A

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

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Please turn over

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Question 11 (2 marks)

Find the centre and radius of the circle with the equation $x^2 + 4x + y^2 - 8y - 5 = 0$. **2**

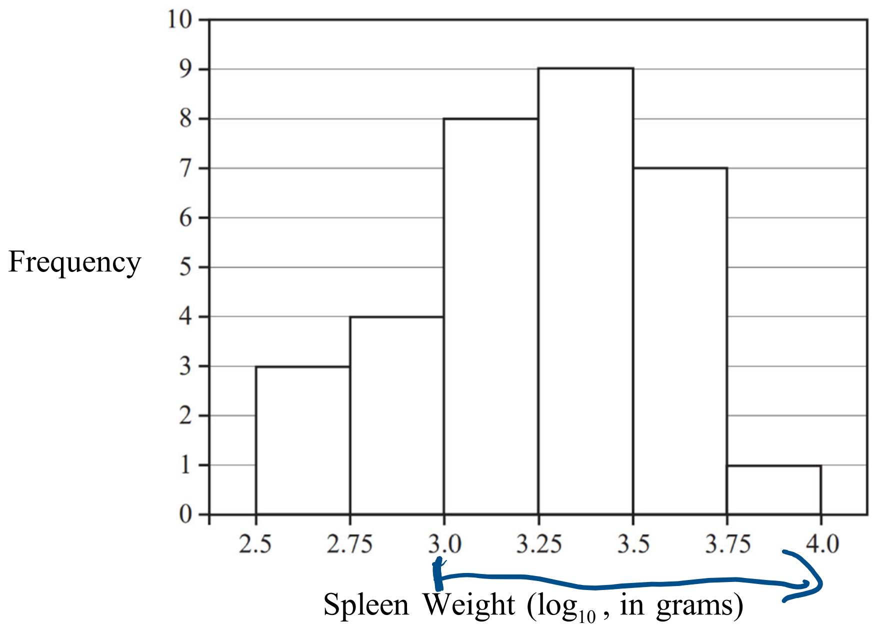
$x^2+4x+4 + y^2-8y+16 = 5+4+16$

$(x+2)^2+(y-4)^2 = 25$

centre $=(-2,4)$ $r=5$

Question 12 (1 mark)

The histogram below shows the distribution of spleen weight for a sample of 32 seals. **1**
The histogram has a $\log_{10}$ scale.



Find the number of seals in this sample with a spleen weight of more than 1000 grams.

$\log_{10}(1000)=3$

$\therefore 8+9+7+1=25$

$\therefore$ 25 seals

Question 13 (5 marks)

The table shows the height and weight of basketball players on the 2013 roster for the NBL Perth Wildcats.

Height H (cm)	191	201	200	204	211	192	196	203	188	202	186
Weight W (kg)	92	99	95	109	105	97	95	100	82	103	92

(a) Using your calculator, find the correlation coefficient for this data correct to two decimal places. **1**

$r = 0.83$

(b) Describe the correlation between height and weight. **1**

Strong, positive correlation

(c) Find the equation of the least-squares regression line for this data. **1**

$y = -59.02 + 0.790x$, $W = -59.02 + 0.790H$

(d) Use the equation in part (c) to estimate the weight of a player that is 198 cm tall. Answer to 1 decimal place. **1**

For $h = 198$, $W = -59.02 + 0.79 \times 198$

$= 97.4$ kg

(e) Can your equation from part (c) be used to make reliable estimates for a player who is 162 cm tall? Justify your response. **1**

Extrapolation is generally not reliable for data that far from the collected data set.

Question 14 (1 mark)

If $\int_2^6 g(x)\,dx = 10$, determine the value of $\int_2^6 \left(g(x)+3\right)\,dx$ given $g(x)>0$. **1**

Translated 3 units up.

$\therefore$ Area of added rectangle $= 3\times 4$
$= 12$

$\therefore \int_2^6 g(x)+3\,dx = 10+12$
$= 22$

Question 15 (3 marks)

The tangent to the graph of $f(x) = x^3 - ax^2 + 1$ at $x = 1$ passes through the origin. **3**

Find the value of a.

$f'(x) = 3x^2 - 2ax$
$f'(1) = 3 - 2a$
$\therefore y = (3-2a)x$ $[y = mx]$

At $x = 1$, $f(1) = 1 - a + 1$
$= 2 - a$

$\therefore 2 - a = 3 - 2a$
$a = 1$

Question 16 (5 marks)

Sophie retires from being a teacher, with a superannuation balance of \$775 320. She decides on receiving a monthly annuity of \$7000, with 7.5 % per annum interest paid on the balance, before the annuity is paid at the end of each month.

The annuity can be modelled by the recurrence relation

$$T_{n+1} = (1+r)T_n - 7000, \quad T_0 = 775320$$

where T_n represents the balance in the superannuation fund after n months.

(a) Show that $r = 0.00625$. **1**

$r = \frac{7.5\%}{12}$

$= 0.00625$

(b) After 3 months, what is the balance remaining in the fund? **2**

$T_1 = (1.00625) \times 775320 - 7000$

$= 773165.75$

$T_2 = T_1 \times 1.00625 - 7000$

$= 770998.04$

$T_3 = T_2 \times 1.00625 - 7000$

$= 768816.77$

$\therefore$ \$768816.77 remaing after 3 months.

(c) Sophie received \$7000 per month for 189 months, in addition to a single payment of \$1000.40 to bring the balance to zero. Determine the interest component of the total amount Sophie received. **2**

Interest = Annuity value − Initial super balance

= (7000 × 189 + 1000.40) − 775320

= 548 680.40

∴ Total Interest received = \$548 680.40

End of Booklet A

Extra Writing Space (if required)

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:			
Set:			

Mathematics Advanced

Section II Answer Booklet **B**

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
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Booklet E — Attempt Questions 30–33 (19 marks)

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Please turn over

Question 17 (7 marks)

Consider a random variable X with a probability density function defined by:

$$f(x) = \begin{cases} kx^2 & 2 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{168}$. **2**

$CDF = k\int_2^8 x^2 \, dx$

$= k\left[\frac{x^3}{3}\right]_2^8$

$= 168k$

Let $CDF = 1$, $168k = 1$

$k = \frac{1}{168}$

(b) Find the mode. **2**

$f(x) = \frac{x^2}{168}$

$f'(x) = \frac{x}{84}$

Let $f'(x) = 0$, $\frac{x}{84} = 0$

$x = 0$ (outside of domain).

$\therefore f(2) = \frac{1}{42}$

$f(8) = \frac{8}{21}$

$\therefore$ The mode is at $x = 8$.

(c) Find the median correct to two decimal places. **2**

$\int_2^m \frac{x^2}{168} dx = \frac{1}{2}$

$\frac{1}{168}\left[\frac{m^3-8}{3}\right] = \frac{1}{2}$

$\frac{m^3-8}{3} = 84$

$m^3 - 8 = 252$

$m^3 = 260$

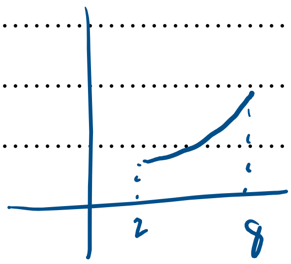
$m = \sqrt[3]{260}$

$= 6.38$

(d) Given the mean is $x = 6.07$, describe the skew of the distribution. Justify your answer. **1**

Since the mean < med (< mode), there is a negative skew.

OR graphic justification



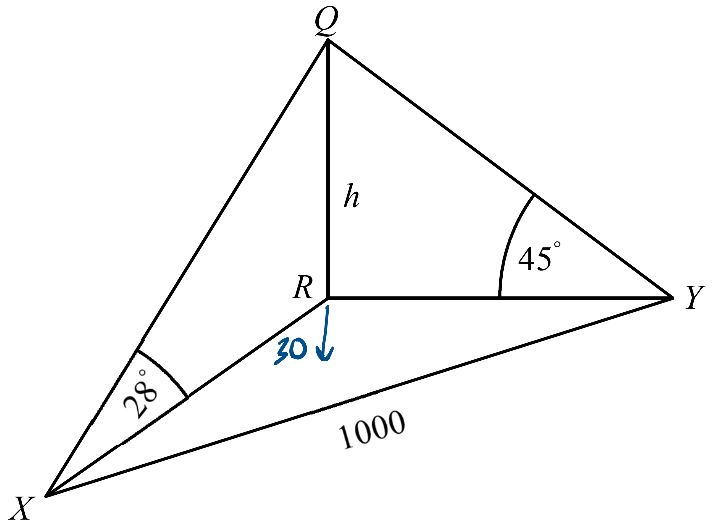
OR mean − median = 6.07 − 6.38

= −0.31 < 0

∴ Negative skew

Question 18 (5 marks)

The angle of elevation of a tower QR of height h metres from a point Y due east of it is 45°. From another point X, the bearing of the tower is 030° and the angle of elevation is 28°. The points X, Y and R are on the same level ground. The distance between X and Y is 1000 metres.



(a) Show that $\angle XRY = 120^\circ$. **1**

$\angle XRY = 30 + 90$

$= 120$

(b) Find an expression for XR. **1**

$\tan 28 = \frac{h}{XR}$

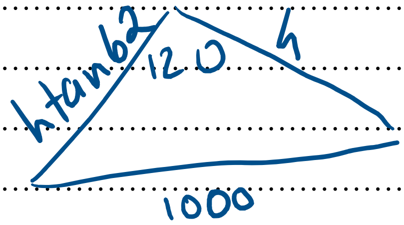
$XR = \frac{h}{\tan 28}$

$= h \cot 28$

$\tan 62 = \frac{XR}{h}$

$XR = h \tan 62$

(c) Hence find the height of the tower QR. Give you answer to the nearest metre. **3**



$$1000^2 = h^2\tan^2 62 + h^2 - 2h^2\tan 62\cos 120$$

$$= h^2(\tan^2 62 + 1 - 2\tan 62\cos 120)$$

$$1000^2 = h^2(6.417...)$$

$$h^2 = \frac{1000^2}{6.417...}$$

$$h = \sqrt{\frac{1000^2}{6.417...}}$$

$$= 394.734..$$

$$= 395 \text{ (nearest metre)}$$

Question 19 (2 marks)

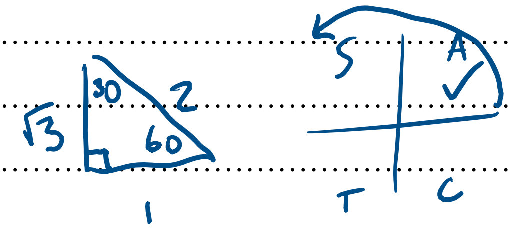
Solve $2 = 4\cos 3x$, in the interval $0 \le x \le \frac{\pi}{4}$. **2**

$\cos 3x = \frac{1}{2}$

$0 \le x \le \frac{\pi}{4}$

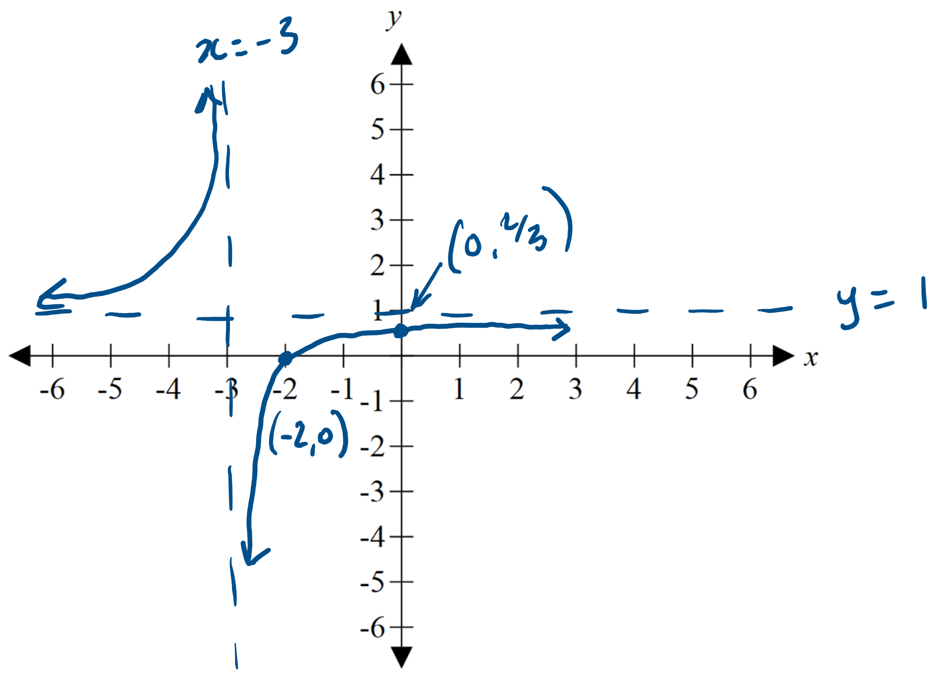
$\therefore 3x = \frac{\pi}{3}$ $\qquad$ $0 \le 3x \le \frac{3\pi}{4}$

$x = \frac{\pi}{9}$



Question 20 (2 marks)

Sketch the graph $y = 1 - \frac{1}{x+3}$, showing asymptotes and the x and y intercepts. **2**



End of Booklet B

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:			
Set:			

Mathematics Advanced

Section II Answer Booklet

C

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
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Booklet C — Attempt Questions 21–25 (18 marks)
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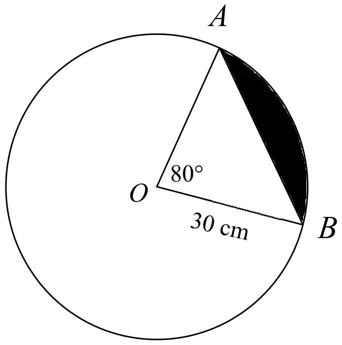
Please turn over

Question 21 (5 marks)

The diagram shows a circle with centre O and radius 30 cm. The points A and B lie on the circle such that $\angle AOB = 80^\circ$.



(a) Convert 80° to radians. **1**

$\frac{4\pi}{9}$

(b) Show that the area of sector AOB is equal to $628\,\text{cm}^2$, rounded to the nearest square centimetre. **1**

$A = \frac{1}{2}(30)^2\left(\frac{4\pi}{9}\right)$

$= 628.3185\ldots$

$\approx 628\ \text{cm}^2$

(c) Find the perimeter of the shaded segment, giving your answer correct to one decimal place. **3**

Arc Length $= r\theta$

$= 30 \times \frac{4\pi}{9}$

$L = \frac{120\pi}{9}$

Chord Length: $\frac{AB}{\sin 80} = \frac{30}{\sin 50}$

$AB = \frac{30 \sin 80}{\sin 50}$

$\therefore$ Perimeter $= \frac{120\pi}{9} + \frac{30 \sin 80}{\sin 50}$

$= 80.5$ cm

Question 22 (3 marks)

Solve the equation $\ln(x^2 - 5) = 2\ln x - \ln 5$. **3**

$\ln(x^2-5) = \ln(x^2) - \ln 5$

$\ln(x^2-5) = \ln\left(\frac{x^2}{5}\right)$

$x^2 - 5 = \frac{x^2}{5}$

$5x^2 - 25 = x^2$

$4x^2 = 25$

$x^2 = \frac{25}{4}$

$x = \pm \frac{5}{2}$ $(\frac{1}{2})$

$\therefore x = \frac{5}{2}$ since $x \neq -\frac{5}{2}$. $(\frac{1}{2})$

Question 23 (5 marks)

Professor Smith has a colony of bacteria. Initially there are 1000 bacteria.

The number of bacteria $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$.

(a) After 20 minutes there are 2000 bacteria. Find the value of k correct to four decimal places. **2**

$2000 = 1000\,e^{20k}$

$2 = e^{20k}$

$\ln(2) = 20k$

$k = \frac{\ln(2)}{20}$

$= 0.0347$

(b) Find the amount of bacteria when $t = 120$. **1**

$N(120) = 1000\,e^{0.0347 \times 120}$

$= 64328.32195$

≈ 64328

Note: If $k = \frac{\ln 2}{120}$ is used, answer is 64000

(c) What is the rate of change of the number of bacteria per minute, when $t = 120$? **2**

$\dot{N}(t) = 1000 \times 0.0347\,e^{0.0347t}$

$= 34.7\,e^{0.0347t}$

$\dot{N}(120) = 34.7\,e^{0.0347 \times 120}$

$= 2218.07$

$\therefore$ Increasing by ~~2218~~ 2183 bacteria per minute

If $k = \frac{\ln 2}{120}$ answer is 2232.2

OR $\dot{N}(120) = kN(120)$

$= 0.0347 \times 64328$

$= \ldots$

Question 24 (3 marks)

600 students at a primary school were asked whether they preferred Rugby, Soccer or AFL. The results are given in the following table.

	Rugby	Soccer	AFL
Female	52	130	88
Male	104	155	71

A student is chosen at random.

(a) Find the probability that the student prefers Rugby. **1**

$P(R) = \frac{13}{50}$

(b) Determine whether the events "the student is Female" and "the student prefers Rugby" are independent, justifying your answer with mathematical reasoning. **2**

$P(F \cap R) = P(F)P(R)$

$LHS = P(F \cap R) = \frac{52}{600}$

$RHS = P(F)P(R) = \frac{270}{600} \times \frac{13}{50} = \frac{117}{1000}$

$\therefore P(F \cap R) \neq P(F)P(R)$

$\therefore$ Not independent

OR $P(F|R) \neq P(F)$

$P(R|F) \neq P(R)$

Question 25 (2 marks)

The graph $y = f(x)$ of $f(x) = x^3$ is translated 3 units right and 5 units up, then horizontally dilated by a scale factor of $\frac{1}{2}$ to produce $y = g(x)$. Find the equation of the transformed function $g(x)$. **2**

$g(x) = (2x - 3)^3 + 5$ ① ½ ½

End of Booklet C

Extra Writing Space (if required)

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:			
Set:			

Mathematics Advanced

Section II Answer Booklet D

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
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Question 26 (4 marks)

Susannah is competing in a card building competition where the number of cards to build each level follows an arithmetic sequence. She builds the fifth level of the stack using 200 cards. For the first four levels, she uses a combined total of 1200 cards.

(a) Find the common difference between the number of cards used to build each level. **2**

$T_5 = 200$, $S_4 = 1200$

$a + 4d = 200$, $\frac{4}{2}(2a + 3d) = 1200$

$a = 200 - 4d$, $2a + 3d = 600$

$\therefore 2(200 - 4d) + 3d = 600$

$400 - 8d + 3d = 600$

$400 - 5d = 600$

$d = -40$

$\therefore$ difference is 40 cards

(b) How many cards were used to create the base level? **1**

If $d = -40$, $T_5 = 200$

$200 = a + 4 \times -40$

$a = 360$

$\therefore$ 360 cards

(c) How many levels will Susannah be able to make? **1**

$T_n = 40$

$40 = 360 + (n-1)(-40)$

$40n - 40 = 320$

$40n = 360$

$n = 9$

$\therefore$ 9 levels

T_n can be any value less than 40

Must reason why n = 9 though.

Question 27 (3 marks)

Show that $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta + \cot\theta} = \cos\theta$. **3**

$$LHS = \frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta}$$

$$= \sin\theta(1+\cot\theta) - \frac{\tan\theta\sec\theta}{\tan^2\theta + 1}$$

(1)

$$= \sin\theta\left(1 + \frac{1}{\tan\theta}\right) - \frac{\tan\theta\sec\theta}{\sec^2\theta}$$

$$= \sin\theta + \cos\theta - \frac{\tan\theta}{\sec\theta}$$

$$= \sin\theta + \cos\theta - \sin\theta$$

$$= \cos\theta$$

$= RHS$ as required

OR

LHS

$$= \frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \sin\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right) - \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$$

Pythag. id. → $\frac{1}{2}$

$$= \sin\theta + \cos\theta - \frac{\frac{\sin\theta\cos\theta}{\cos\theta}}{1}$$

$$= \sin\theta + \cos\theta - \sin\theta$$

$$= \cos\theta$$

$= RHS$

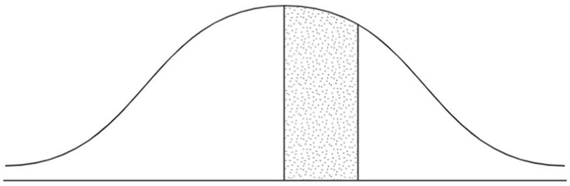
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Question 28 (6 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table below shows the probability this random variable lies between 0 and z for different values of z.

z	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



(a) Using the table, find the probability that a value lies between 0.2 and 0.6. **1**

$P(0.2 < z < 0.6) = 0.2257 - 0.0793$

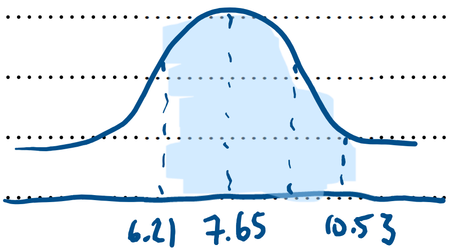
$= 0.1464$

Daily charges for gas usage are normally distributed with a mean of \$7.65 and standard deviation of \$1.44.

(b) Two adults are comparing their gas charges. Find the probability that both of their gas charges are between \$6.21 and \$10.53. **2**

$68\% + 13.5\% = 81.5\%$

$\therefore$



$P(\text{both}) = 0.815 \times 0.815$

$= 0.664225$

$= 66.4\%$

(c) By first calculating a z-score, find how many people out of 1000, are expected to have a daily charge greater than \$8.37. **3**

$\mu = 7.65 \qquad \sigma = 1.44$

$z = \dfrac{8.37 - 7.65}{1.44}$

$= 0.5$ ← (1)

$P(z > 0.5) = 0.5 - 0.1915$ (OR) $= 1 - (0.5 + 0.1915)$ (½)

$= 0.3085$ $= 0.3085$

$\therefore 1000 \times 0.3085 = 308.5$ (½)

$\therefore$ 308 or 309 people

(1) (accept both)

including error carried forward (ecf)

Question 29 (7 marks)

Consider the curve $y = 3x^2 + x^3$.

(a) Find the coordinates of any stationary point(s) and point(s) of inflection then determine their nature. **4**

$y' = 6x + 3x^2$

Let $y' = 0$, $6x + 3x^2 = 0$

$3x(2 + x) = 0$

$\therefore x = 0, x = -2$

x	-3	-2	-1	0	1
y'	9	0	-3	0	9
	/	—	\	—	/

$\therefore (0,0)$ is a min T.P

$(-2, 4)$ is a max T.P

$y'' = 6 + 6x$

Let $y'' = 0$, $6 + 6x = 0$

$x = -1$

x	-1.5	-1	-0.5
y''	-3	0	3
	∩	—	∪

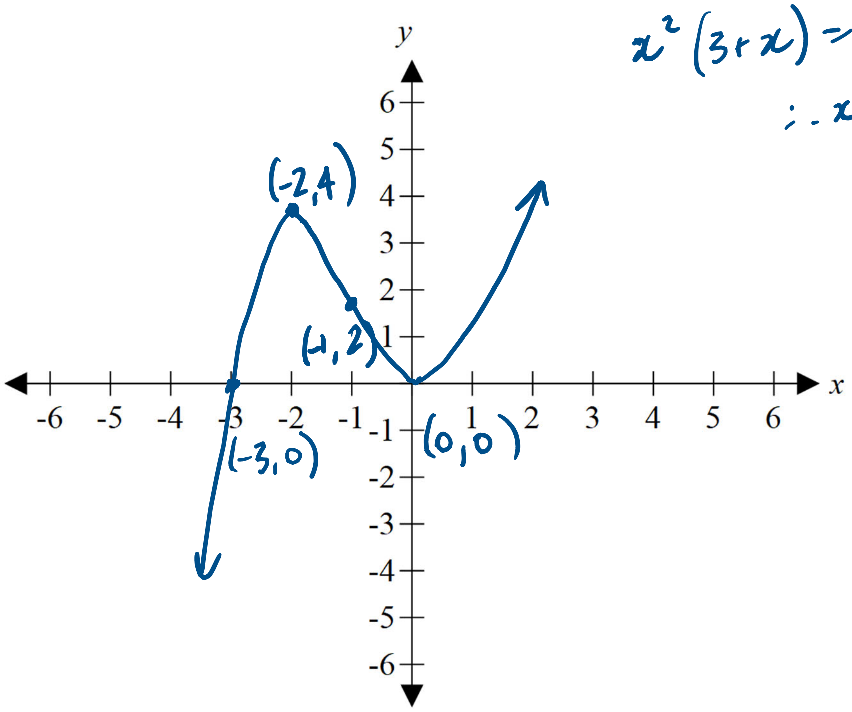
$\therefore$ change in concavity

$\therefore (-1, 2)$ is a point of inflection

(b) Sketch the curve $y = 3x^2 + x^3$, showing any stationary points, points of inflection and intercepts with the axes. **2**

$x^2(3+x)=0$

$\therefore x=0,\ x=-3$



(c) For what interval is the curve $y = 3x^2 + x^3$ decreasing? **1**

$-2 < x < 0$, $x \in (-2, 0)$

End of Booklet D

Extra Writing Space (if required)

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2023 YEAR 12 TRIAL EXAMINATION

Examination Number:	
Set:	

Mathematics Advanced

Section II Answer Booklet E

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Booklet A — Attempt Questions 11–16 (17 marks)
Booklet B — Attempt Questions 17–20 (16 marks)
Booklet C — Attempt Questions 21–25 (18 marks)
Booklet D — Attempt Questions 26–29 (20 marks)
Booklet E — Attempt Questions 30–33 (19 marks)

Instructions

- Write your Examination Number and Set at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on the back of the booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 30 (3 marks)

A particle moves in a straight line with an initial displacement equal to 1 metre and initial velocity equal to $1\ \mathrm{m\,s^{-1}}$. Find the exact position of the particle after 5 seconds if its acceleration in $\mathrm{m\,s^{-2}}$ is given by $a = 2\sin t$. Give your answer to the nearest metre. **3**

$v = \int 2\sin t\, dt$

$v = -2\cos t + c$

Let $t=0$ & $v=1$, $1 = -2\cos(0) + c$

$c = 3$

$v = -2\cos t + 3$

$d = \int -2\cos t + 3\, dt$

$d = -2\sin t + 3t + e$

Let $t=0$ & $d=1$, $1 = -2\sin(0) + 3(0) + e$

$\therefore e = 1$

$\therefore d = -2\sin t + 3t + 1$

At $t=5$, $d = -2\sin(5) + 3(5) + 1$

$= 17.9\ldots$

$\therefore$ Approximately 18m to the right of the origin

Question 31 (4 marks)

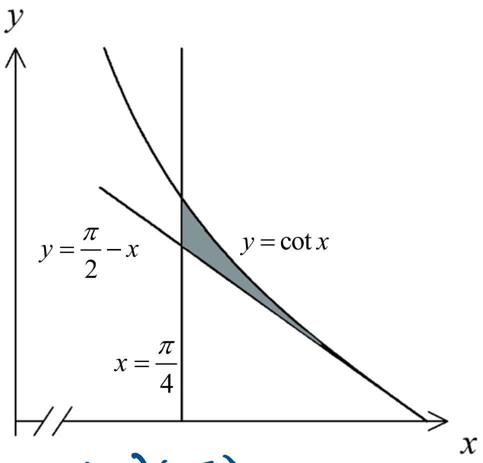
(a) Show that $\frac{d}{dx}\left(\log_e(\sin x)\right) = \cot x$. **1**

$\frac{d}{dx} \log_e(\sin x) = \frac{1}{\sin x} \times \cos x$

$= \frac{\cos x}{\sin x}$

$= \cot x$

(b) The shaded region in the diagram is bounded by the curve $y = \cot x$ and the lines $y = \frac{\pi}{2} - x$ and $x = \frac{\pi}{4}$. Using the result of part (a), or otherwise, find the exact area of the shaded region. **3**



$\int_{\pi/4}^{\pi/2} \cot x \, dx - \frac{1}{2}\left(\frac{\pi}{4}\right)\left(\frac{\pi}{4}\right)$

$= \left[\ln(\sin x)\right]_{\pi/4}^{\pi/2} - \frac{\pi^2}{32}$

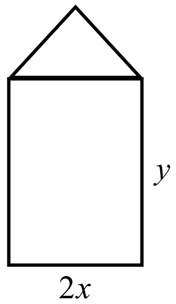
$= \ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{4}\right) - \frac{\pi^2}{32}$

$= \ln(1) - \ln\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi^2}{32}$

$= \ln(\sqrt{2}) - \frac{\pi^2}{32}$ OR $\ln(2^{1/2}) - \frac{\pi^2}{32}$ OR $\frac{1}{2}\ln(2) - \frac{\pi^2}{32}$

Question 32 (5 marks)

A stained glass feature is being created in the shape of a rectangle surmounted by an isosceles triangle of height equal to half its base. The perimeter is to be 150 cm. **5**



By showing that the area, in square centimetres, of the stained glass figure is given by $A = 150x - (2\sqrt{2}+1)x^2$. Determine the width and the height of the figure for which the area is the greatest. Give your answer to one decimal place.

$P = 2x + y + y + \sqrt{2}x + \sqrt{2}x$

$150 = 2x + 2y + 2\sqrt{2}x$

$75 = x + y + \sqrt{2}x$

$y = 75 - x - \sqrt{2}x$

$\therefore A = 2xy + \frac{1}{2} \times 2x \times x$

$= 2x(75 - x - \sqrt{2}x) + x^2$

$= 150x - 2x^2 - 2\sqrt{2}x^2 + x^2$

$= 150x - x^2 - 2\sqrt{2}x^2$

$= 150x - (1 + 2\sqrt{2})x^2$

$\therefore \frac{dA}{dx} = 150 - 2(1 + 2\sqrt{2})x$

Let $\frac{dA}{dx} = 0$

$0 = 150 - 2(1 + 2\sqrt{2})x$

$2(1 + 2\sqrt{2})x = 150$

$$x = \frac{150}{2(1+2\sqrt{2})}$$

$$= 19.6$$

x	19	19.6	20
A'	4.5	0	-3.1
	/	—	\

$\therefore x = 19.6$ is a maximum.

$\therefore$ Width $= 2x$

$= 2(19.6)$

$= 39.2$ cm

Height $= y + x$

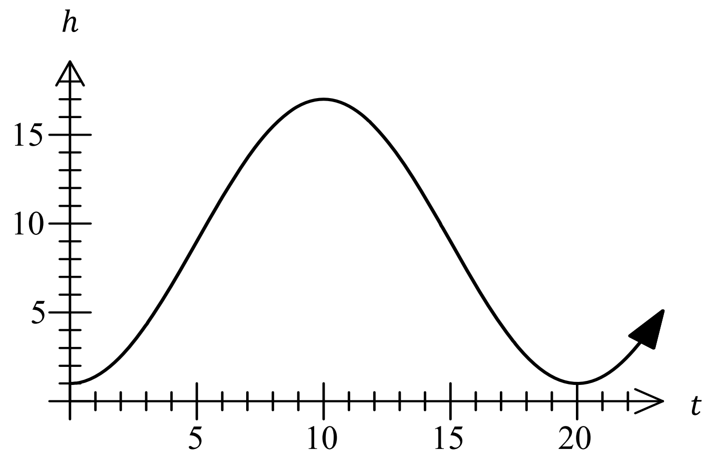
$= 19.6 + (75 - (1+\sqrt{2}) \times 19.6)$

$= 47.3$ cm

Question 33 (7 marks)

On a Ferris Wheel at a fair, the height of a carriage from the ground is modelled by the function $h(t) = -a\cos\left(\frac{\pi t}{10}\right) + b$, where t is the number of seconds after the ride has started, h is the height in metres and a and b are constants. The carriage has a maximum height of 17 metres when $t = 10$ and a minimum height of 1 metre when $t = 20$.

The graph of $h(t)$ is shown.



(a) What are the values of a and b? **2**

$a = \frac{17-1}{2}$ $\quad$ $b = 1+8$

$= 8$ $\quad$ $= 9$

(b) If the ride lasts for 3 minutes. How many rotations are completed before the ride comes to a stop? **2**

Period = 20 sec (from graph)

$\therefore 180 \div 20 = 9$

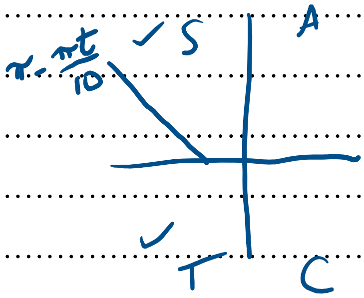
$\therefore$ 9 rotations

(c) When the carriage is 11 metres or higher above the ground passengers can see the ocean in the distance. On a 3-minute ride, for how long do they see the ocean to the nearest second? **3**

$-8\cos\left(\frac{\pi t}{10}\right)+9=11$

$-8\cos\left(\frac{\pi t}{10}\right)=2$

$\cos\left(\frac{\pi t}{10}\right)=-\frac{1}{4}$

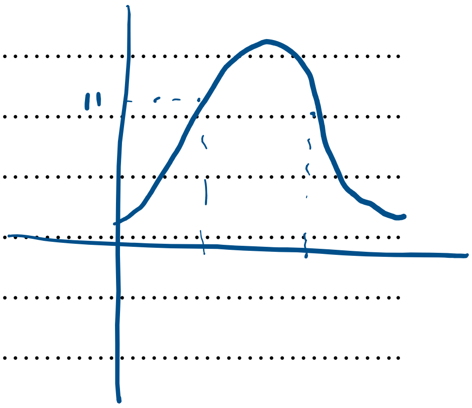


$\therefore \pi-\frac{\pi t}{10}=\cos^{-1}\left(-\frac{1}{4}\right)$

$\frac{\pi t}{10}=-\left(\cos^{-1}\left(-\frac{1}{4}\right)-\pi\right)$

$t=\frac{-10\left(\cos^{-1}\left(-\frac{1}{4}\right)-\pi\right)}{\pi}$

$=4.1956\ldots$



$\therefore 2\times 4.1956=8.3912$

$\therefore$ 8.3912 seconds per rotation

$\therefore 9\times 8.3912=75.52$ seconds of the 3 minute ride.

End of paper